ME 472 – Engineering Metrology and Quality Control

Chp 4 - Statistical Process Control (SPC)

Mechanical Engineering
University of Gaziantep

Dr. A. Tolga Bozdana
Assistant Professor

Statistical Process Control (SPC) & Control Charts

Definition of SPC

- SPC uses statistical tools to **observe the performance of production process** in order to predict significant deviations that may later result in rejected product.
- To control a process based on varying data, it is necessary to keep a check on the current state of **accuracy (central tendency)** and **precision (spread)** of the distribution of data.

Achievement of SPC

- SPC is achieved with the aid of **control charts** *(pioneered by Walter A. Shewhart in 1920)*.
- The most frequently used charts are **Mean and Range Charts**, which are used together.

Implementation of Control Charts

- The operation of control charts to detect the state of control of a process is as follows:

– Periodically, **samples of a given size** (*four steel rods, five tins of paint, eight tablets, four delivery times, etc.*) are taken from the process at reasonable intervals, when it is believed to be stable or in-control and adjustments are not being made.

– **The variable** (*length, volume, weight, time, etc.*) is measured for each item of the sample so that the sample mean and range values are recorded on a chart.

– **Control limits (i.e. warning & action limits)** are determined to check stability of the process.

– If the process is not in control, efforts are made for making the process under control.

– Once process is in control, its capability for specified tolerances is examined by **capability indices**.

Sample No.	Rod Lengths (mm) i	ii	iii	iv
1	144	146	154	146
2	151	150	134	153
3	145	139	143	152
4	154	146	152	148
5	157	153	155	157
6	157	150	145	147
7	149	144	137	155
8	141	147	149	155
9	158	150	149	156
10	145	148	152	154
11	151	150	154	153
12	155	145	152	148
13	152	146	152	142
14	144	160	150	149
15	150	146	148	157
16	147	144	148	149
17	155	150	153	148
18	157	148	149	153
19	153	155	149	151
20	155	142	150	150
21	146	156	148	160
22	152	147	158	154
23	143	156	151	151
24	151	152	157	149
25	154	140	157	151

Suppose that, we have **100 rod lengths (x)** as **25 samples (k)** of **size 4 (n)** were taken.

Lets represent this data using **a histogram chart**, which is used for showing the frequency of data values. First, we must define **the bins (i.e. intervals)**.

For this purpose, we search for **the lowest** and **the highest** values in the data set (i.e. **134** and **160**).

Then, we define starting & end values for the bins (i.e. **133.5** and **160.5**).

After this, we divide the difference between these values by equal intervals (i.e. **27/3** which gives **9 equal intervals**).

133.5 – 136.5
136.5 – 139.5
139.5 – 142.5
142.5 – 145.5
145.5 – 148.5
148.5 – 151.5
151.5 – 154.5
154.5 – 157.5
157.5 – 160.5

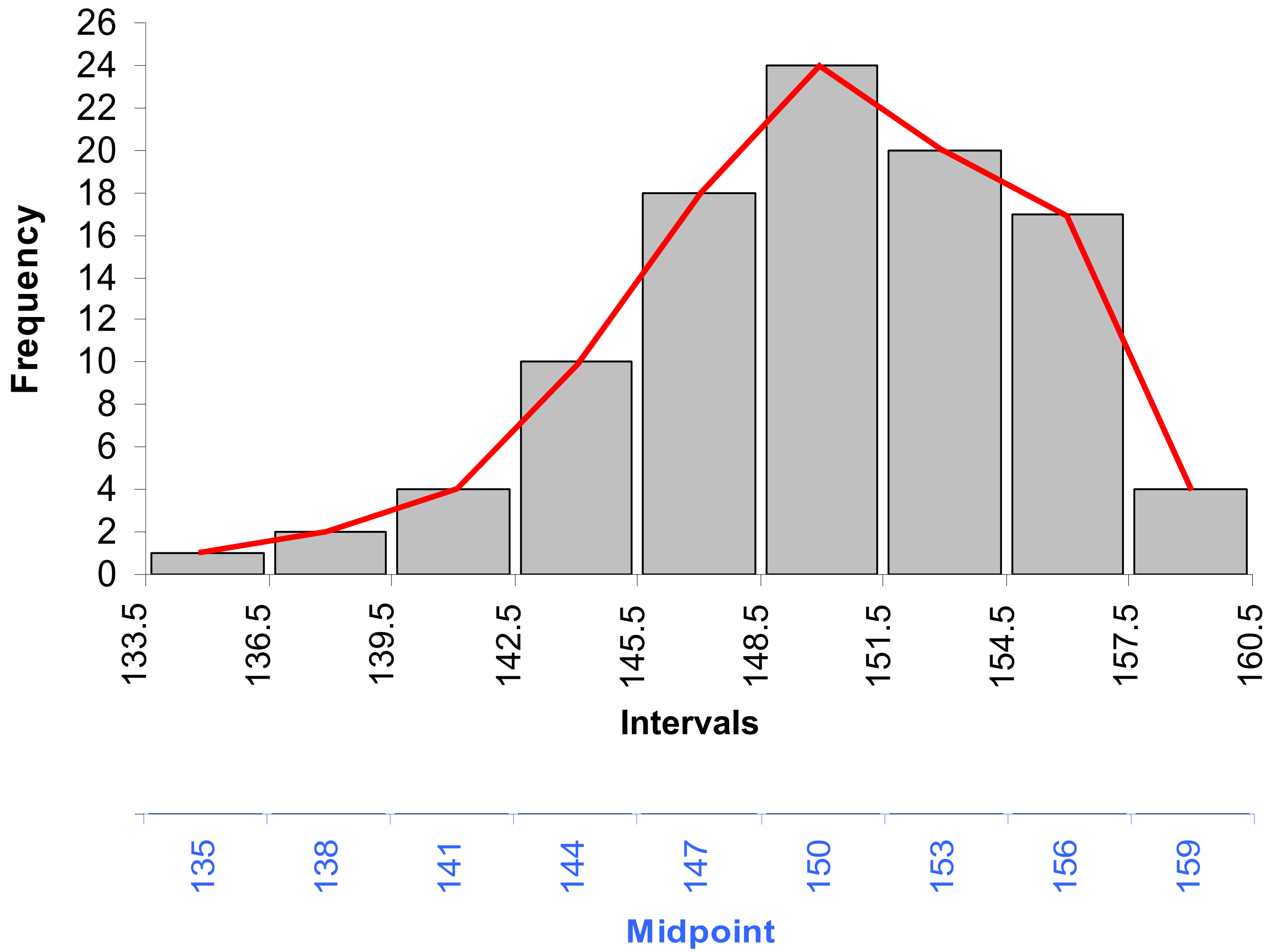
Histogram Chart

The histogram chart is obtained using **Microsoft Excel** *(Click on the **"Data Analysis"** under "Tools" menu and choose **"Histogram"** option from the list. If you can't see "Data Analysis" function, then go to **"Add-ins"** under "Tools" menu and click on **"Analysis Toolpak"**)*.

Select the data set and the intervals so that the histogram chart will be built. The frequency of each value within the data set according to each bin will be calculated automatically.

Sometimes, it is more practical to show midpoints on the chart instead of intervals. In such cases, midpoint of each bin is calculated (e.g. mean of interval 133.5 – 136.5 is 135).

Histogram charts are also useful for showing the distribution of data sets.



Sample Mean & Grand (Process) Mean

Sample No.	Rod Lengths (mm) i	ii	iii	iv	Sample Mean (mm)
1	144	146	154	146	147.50
2	151	150	134	153	147.00
3	145	139	143	152	144.75
4	154	146	152	148	150.00
5	157	153	155	157	155.50
6	157	150	145	147	149.75
7	149	144	137	155	146.25
8	141	147	149	155	148.00
9	158	150	149	156	153.25
10	145	148	152	154	149.75
11	151	150	154	153	152.00
12	155	145	152	148	150.00
13	152	146	152	142	148.00
14	144	160	150	149	150.75
15	150	146	148	157	150.25
16	147	144	148	149	147.00
17	155	150	153	148	151.50
18	157	148	149	153	151.75
19	153	155	149	151	152.00
20	155	142	150	150	149.25
21	146	156	148	160	152.50
22	152	147	158	154	152.75
23	143	156	151	151	150.25
24	151	152	157	149	152.25
25	154	140	157	151	150.50

For the given data, it is often not meaningful to calculate the mean of whole population (i.e. the total output from process rather than a sample):

$$\mu = 150.1mm$$

Instead, **Grand (Process) Mean** (i.e. the mean of all sample means) is a good estimate of population mean:

$$\bar{\bar{X}} = \sum_{i=1}^{k} \bar{X}_i / k$$

For this purpose, the mean of each sample is calculated. For example, the mean of **sample no. 9** is:

$$\bar{X}_9 = (158 + 150 + 149 + 156)/4 = 153.25\,mm$$

Then, the process mean is calculated as:

$$\bar{\bar{X}} = \frac{(147.50 + 147.00 + 144.75 + \ldots + 150.50)}{25} = 150.1mm$$

Based on the information in previous slides, it is now **better to use "the standard deviation of sample means" instead of using "the standard deviation of the population"**.

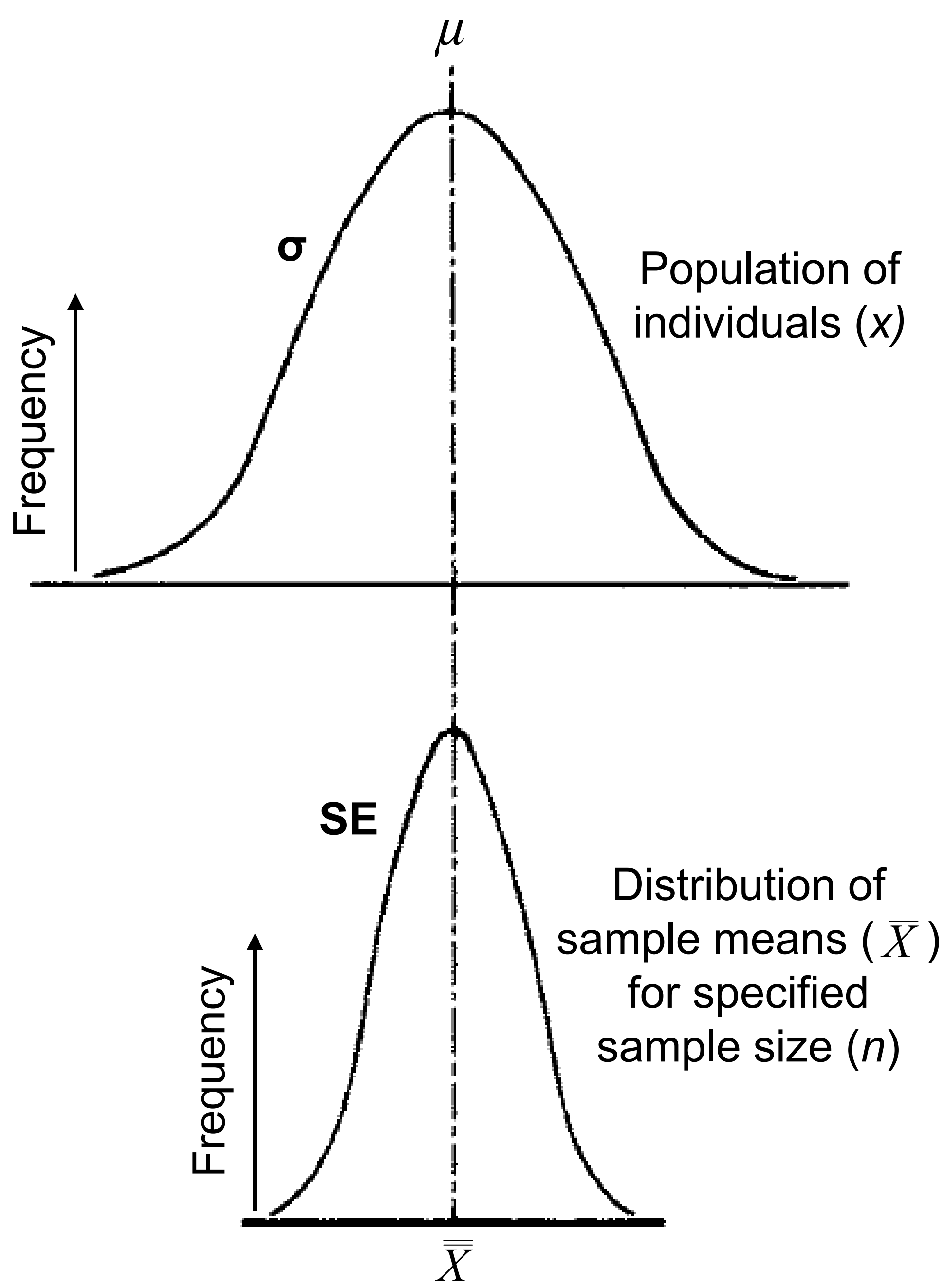
Standard Error (SE) of Means is defined as:

$$SE = \sigma / \sqrt{n}$$

where **σ** is the standard deviation of population, **n** is the sample size (i.e. total number of samples). *In case of our example:* ***n = 4***

SE of means is more reliable than *SD of whole population*. As seen from the graph, *the scatter (i.e. the precision)* of the sample means is **much less** than the scatter of the individual rod lengths. On the other hand, *the mean (i.e. the accuracy)* **remains unchanged** as population is the same:

$$\mu = \overline{\overline{X}} = 150.1\, mm$$



Range (Measure of Precision or Spread)

Sample No.	Rod Lengths (mm) i	ii	iii	iv	Sample Mean (mm)	Sample Range (mm)
1	144	146	154	146	147.50	10
2	151	150	134	153	147.00	19
3	145	139	143	152	144.75	13
4	154	146	152	148	150.00	8
5	157	153	155	157	155.50	4
6	157	150	145	147	149.75	12
7	149	144	137	155	146.25	18
8	141	147	149	155	148.00	14
9	158	150	149	156	153.25	9
10	145	148	152	154	149.75	9
11	151	150	154	153	152.00	4
12	155	145	152	148	150.00	10
13	152	146	152	142	148.00	10
14	144	160	150	149	150.75	16
15	150	146	148	157	150.25	11
16	147	144	148	149	147.00	5
17	155	150	153	148	151.50	7
18	157	148	149	153	151.75	9
19	153	155	149	151	152.00	6
20	155	142	150	150	149.25	13
21	146	156	148	160	152.50	14
22	152	147	158	154	152.75	11
23	143	156	151	151	150.25	13
24	151	152	157	149	152.25	8
25	154	140	157	151	150.50	17

It is possible to simplify the calculation of control limits by using an alternative measure of spread of process: **the mean range of samples**

The range (i.e. difference between the highest and the lowest observations) is the simplest possible measure of scatter:

$$\overline{R} = \sum_{i=1}^{k} R_i / k$$

For instance, the range in **sample no. 9** is:

$$R_9 = 158 - 149 = 9\,mm$$

Hence, **the mean range** (i.e. the mean of all sample ranges) is found as:

$$\overline{R} = \frac{(10 + 19 + 13 + \ldots + 17)}{25} = 10.8\,mm$$

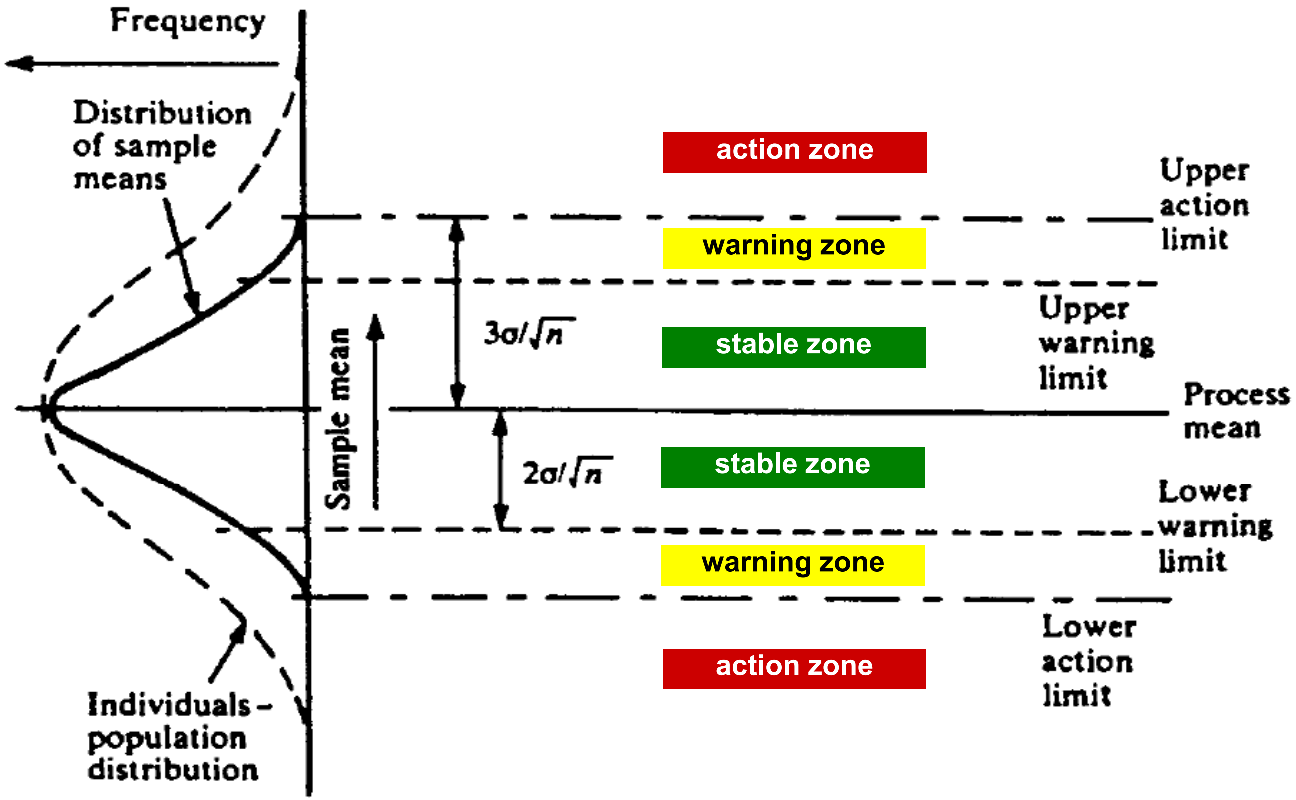
First of all, we draw **the distribution of sample means** graph, and then turn this bell-shape graph onto its side having **process mean in x-axis** and **sample mean in y-axis**.

In a stable process, most of the sample means lie within the range of $\overline{\overline{X}} \pm 3SE$. Thus, we extrapolate the process mean by $\pm 3SE$ on both sides to obtain **"upper and lower action limits"**. If the process is running satisfactorily, we expect from normal distribution that *at least 0.27% (≈ 1 in 370)* of the means of successive samples *might lie out of action limits*.

Extrapolating the process mean by $\pm 2SE$ gives **"upper and lower warning limits"**. *At least 4.45% (≈ 1 in 22)* of the means of successive samples *might lie out of limits*.

The samples between warning limits are said to be reliable **(stable zone)**.

Chance of two consecutive sample means in warning zone is 1/22 x 1/22 = 1/484, which is even lower than chance of one value in action zone. This suggests that **the process is out of control**.



Determination of Action and Warning Limits

Standard deviation can be expressed in terms of the mean range and **Hartley's constant** (**d_n**) :

$$\sigma = \overline{R} / d_n$$

⇩

When the limits are being calculated, the mean range can be used instead of SE:

$$\text{Action Limits} = \overline{\overline{X}} \pm 3SE = \overline{\overline{X}} \pm 3\frac{\sigma}{\sqrt{n}} = \overline{\overline{X}} \pm \frac{3}{d_n\sqrt{n}}\overline{R}$$

$$\text{Warning Limits} = \overline{\overline{X}} \pm 2SE = \overline{\overline{X}} \pm 2\frac{\sigma}{\sqrt{n}} = \overline{\overline{X}} \pm \frac{2}{d_n\sqrt{n}}\overline{R}$$

⇩

d_n and **n** are constants for the same sample size, so it is possible to replace all with one constant: **A_2**

$$\frac{3}{d_n\sqrt{n}} = A_2 \quad \& \quad \frac{2}{d_n\sqrt{n}} = (2/3)A_2$$

⇩

As a result, action and warning limits can be written in terms of the mean range and a constant:

$$\text{Warning Limits} = \overline{\overline{X}} \pm (2/3)A_2\overline{R} \quad \& \quad \text{Action Limits} = \overline{\overline{X}} \pm A_2\overline{R}$$

The constants (**d_n** & **A_2**) for sample sizes (**n**) from 2 to 12 are given in table.

For the sample sizes up to 12, the range method of estimating action and warning limit is relatively efficient.

For the values greater than 12, the range loses its efficiency rapidly since it ignores all the information in the sample between the highest and the lowest values.

The sample sizes of 4 or 5 are generally employed in control charts, which gives satisfactory results.

In case of our example, the sample size (**n**) is 4. Thus, the constants (**d_n** & **A_2**) are selected accordingly.

Sample Size (n)	Hartley's Constant (d_n)	Constant for use with Sample Range	
		A_2	2/3 A_2
2	1.128	1.88	1.25
3	1.693	1.02	0.68
4	2.059	0.73	0.49
5	2.326	0.58	0.39
6	2.534	0.48	0.32
7	2.704	0.42	0.28
8	2.847	0.37	0.25
9	2.970	0.34	0.20
10	3.078	0.31	0.21
11	3.173	0.29	0.19
12	3.258	0.27	0.18

The mean chart is constructed by flipping the bell-shape histogram chart onto its side, hence we have **sample mean as y-axis** and **the process mean as x-axis** *(which was calculated before as 150.1 mm)*.

⇩

Then, action and warning limits are determined based on mean range and constants:

Mean Range (calculated): $\bar{R} = 10.8\,mm$

Hartley's Constant (from table): $\mathrm{d_n} = 2.059$

Sample Range Constants (from table):

$\mathrm{A_2} = 0.73$ & $(2/3)\mathrm{A_2} = 0.49$

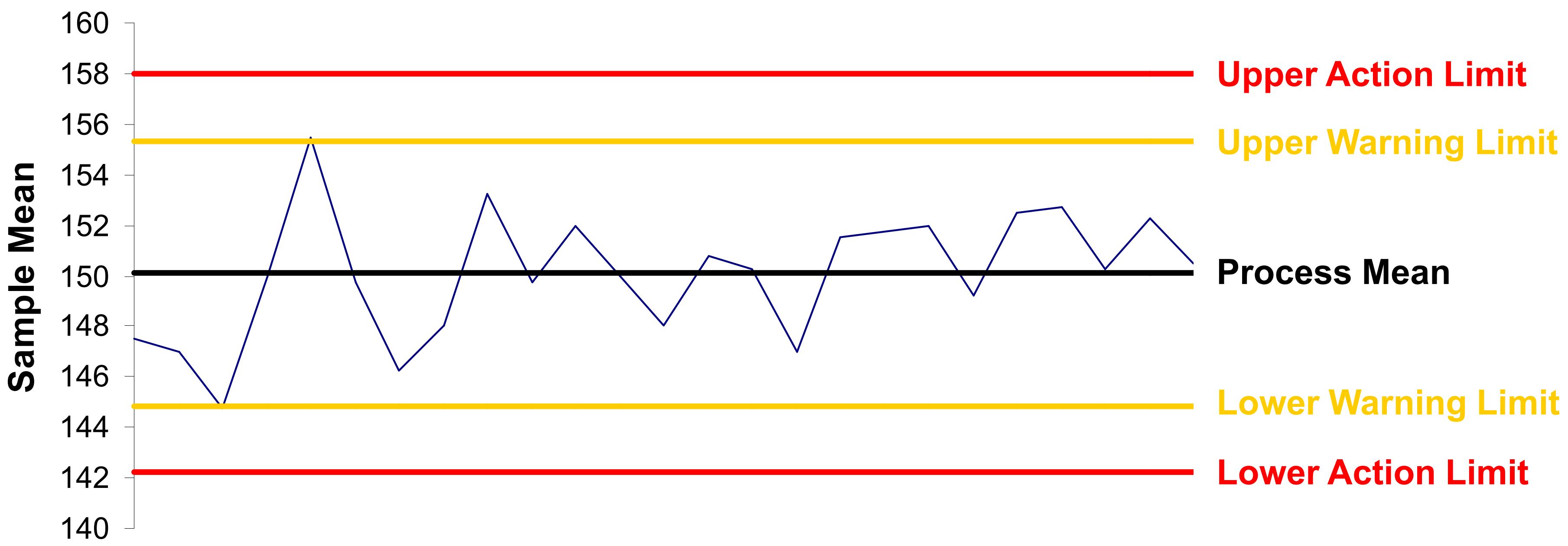
⇩

Upper Warning Limit: $\bar{\bar{X}} + (2/3)\,A_2\bar{R} = 155.40\,mm$

Lower Warning Limit: $\bar{\bar{X}} - (2/3)\,A_2\bar{R} = 144.81\,mm$

Upper Action Limit: $\bar{\bar{X}} + A_2\bar{R} = 157.98\,mm$

Lower Action Limit: $\bar{\bar{X}} - A_2\bar{R} = 142.22\,mm$



Building R-bar (Range) Chart

The control limits on a range chart are asymmetrical about the mean range as the distribution of sample ranges is a positively skewed distribution.

Thus, upper and lower action limits are calculated for **0.1% & 99.9%** of sample range while upper and lower warning limits lie at **2.5% & 97.5%**:

$$\text{Upper Action Limit}: D'_{0.001}\overline{R}$$

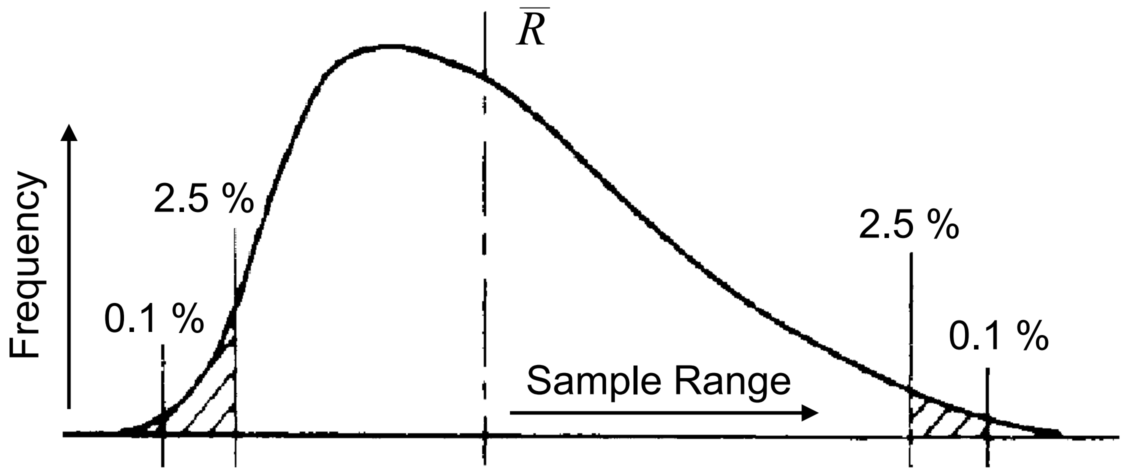
$$\text{Lower Action Limit}: D'_{0.999}\overline{R}$$

$$\text{Upper Warning Limit}: D'_{0.025}\overline{R}$$

$$\text{Lower Warning Limit}: D'_{0.975}\overline{R}$$

The constants for calculating control limits are available for sample sizes (**n**) up to 12 (as given in table).

For our case, constants are selected from the table for sample size of 4.



Sample Size (n)	Constant for use with Mean Range			
	$D'_{0.999}$	**$D'_{0.001}$**	**$D'_{0.975}$**	**$D'_{0.025}$**
2	0.00	4.12	0.04	2.81
3	0.04	2.98	0.18	2.17
4	0.10	2.57	0.29	1.93
5	0.16	2.34	0.37	1.81
6	0.21	2.21	0.42	1.72
7	0.26	2.11	0.46	1.66
8	0.29	2.04	0.50	1.62
9	0.32	1.99	0.52	1.58
10	0.35	1.93	0.54	1.56
11	0.38	1.91	0.56	1.53
12	0.40	1.87	0.58	1.51

The range chart is constructed by turning the bell-shape histogram chart onto its side, hence we have **sample range as y-axis** and **mean range as x-axis** *(which was calculated before as 10.8 mm)*.

Then, action and warning limits are determined based on mean range and constants:

$$D'_{0.001} = 2.57$$
$$D'_{0.999} = 0.10$$
$$D'_{0.025} = 1.93$$
$$D'_{0.975} = 0.29$$

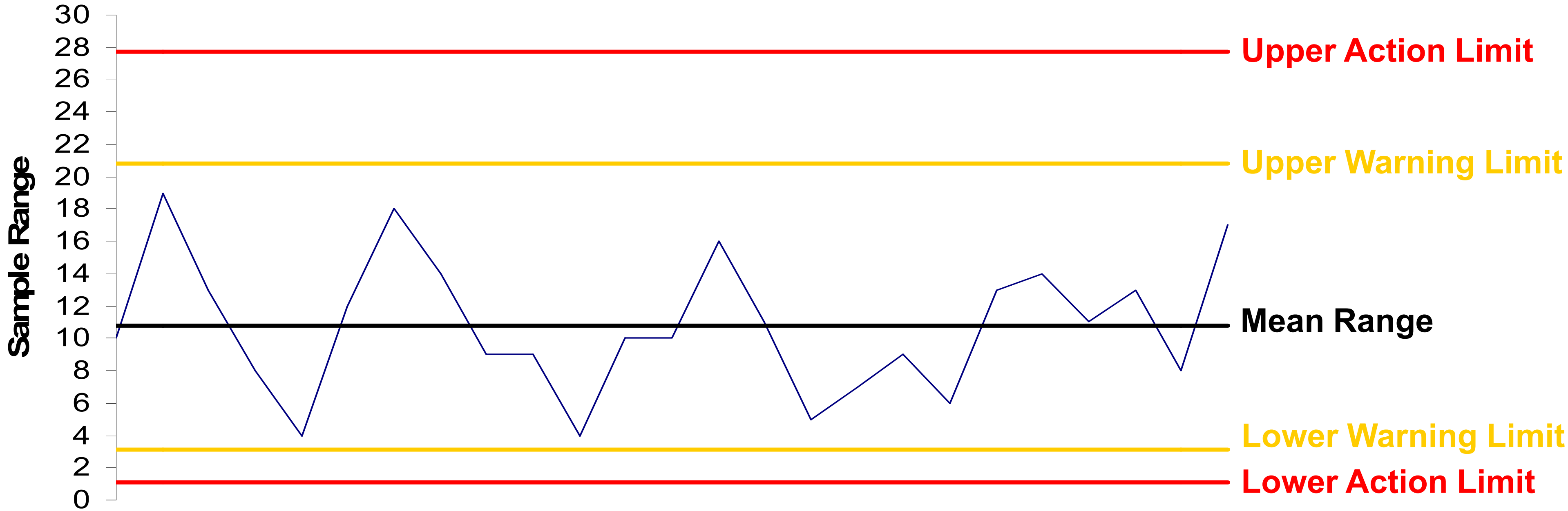
from table

Upper Warning Limit: $D'_{0.025}\overline{R} = 20.8\,mm$

Lower Warning Limit: $D'_{0.975}\overline{R} = 3.1\,mm$

Upper Action Limit: $D'_{0.001}\overline{R} = 27.8\,mm$

Lower Action Limit: $D'_{0.999}\overline{R} = 1.1\,mm$

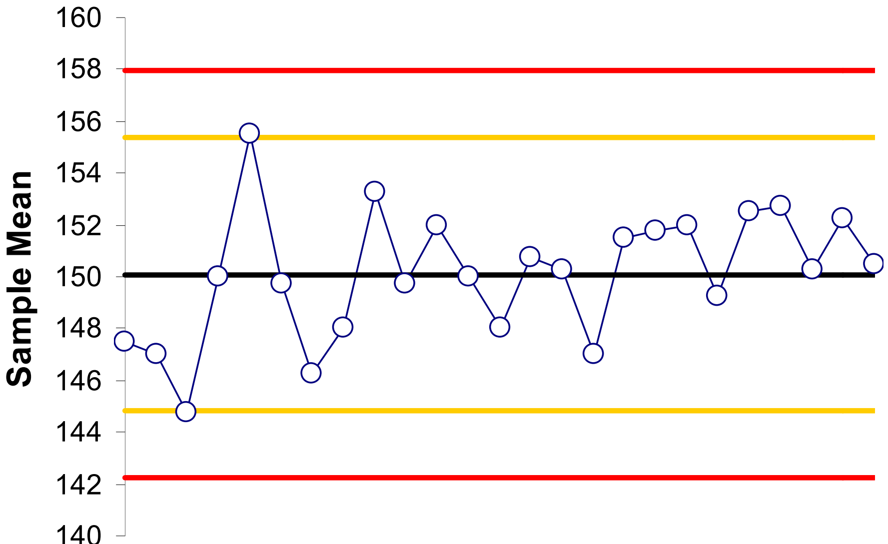


Is the process in control?

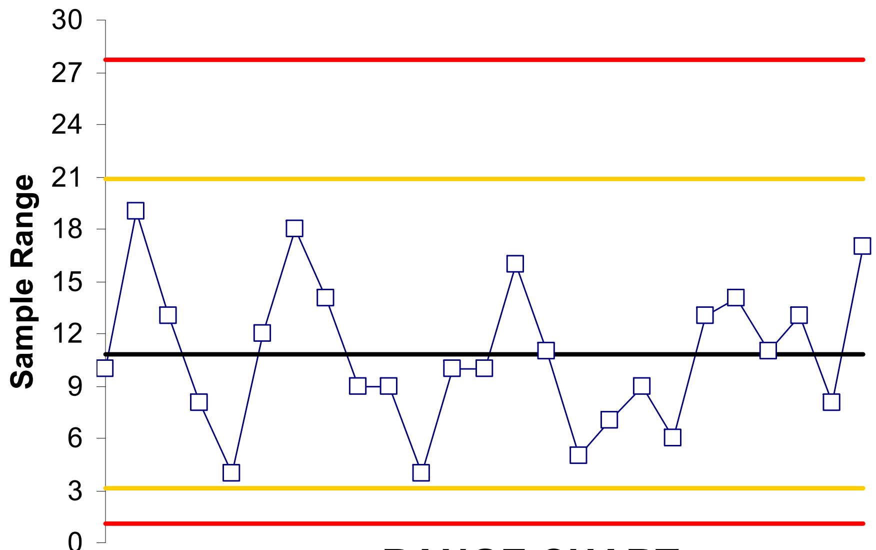
- ✓ NO mean or range values which lie outside the action limits (**action zone**)?
- ✓ NO more than about 1 in 22 values between warning and action limits (**warning zone**)?
- ✓ NO incidence of two consecutive mean or range values which lie outside the same warning limit on either mean or range chart (**warning zone**)?
- ✓ NO run or trend of five or more consecutive mean or range values, which also infringes a warning or action limit (**warning zone** and/or **action zone**)?
- ✓ NO run of more than six sample means which lie either above or below the process mean (**stable zone**)?
- ✓ NO trend of more than six values of the sample means which are either rising or falling (**stable zone**)?

⇩

PROCESS IS IN CONTROL !!!



MEAN CHART



RANGE CHART
(should be examined first)

What if the process is out of control?

Do not use the control charts and investigate the assignable causes of variation.

⇩

The assignable causes of variation have been identified, and now to be eliminated.

Another set of samples from the process is taken and control limits are recalculated.

Approximate control limits are recalculated by simply **excluding the out of control results** for which special causes have been found and corrected.

⇩

The exclusion of samples representing unstable conditions is not just throwing away bad data. By excluding the data affected by known causes, we have a better estimate of variation due to common causes only.

The process is re-examined to see if it is in statistical control.

⇩

If the process is shown to be in statistical control, the next task is to **compare the limits** of this control **with the required tolerance**.

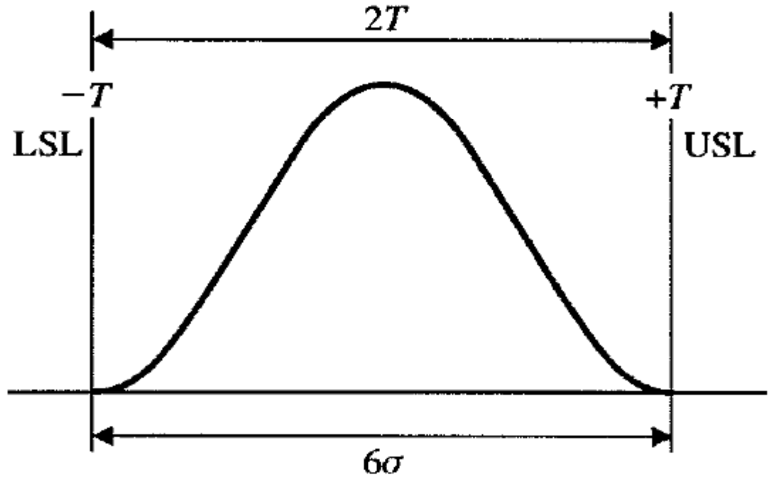
Control Limits vs. Tolerance Limits

- **Tolerance limits** should be based on **the functional requirements of the product**.
  Limits on control charts are based on **the stability and actual capability of the process**.
- A process may not meet the specification requirements, but still be in a state of statistical control.
- A comparison of process capability and tolerance can only take place, with confidence, when the process is statistically in control. Thus, in controlling a process, it is necessary **to establish first that it is in statistical control and then to compare** its centring & spread with the specified target value and the specification tolerance.

The relationship between process variability and tolerances can be formalized by consideration of standard deviation of the process. In order to manufacture within the specification, distance between **Upper Specification Limit (USL)** or **Upper Tolerance (+T)** and **Lower Specification Limit (LSL)** or **Lower Tolerance (–T)** must be analyzed.

There are three precision levels:

- **High Relative Precision:** $2T \gg 6\sigma$
- **Medium Relative Precision:** $2T \geq 6\sigma$
- **Low Relative Precision:** $2T < 6\sigma$



Process Capability Index

- It is **a measure relating the actual performance of a process to its specified performance**, where the processes are considered to be a combination of plant or equipment, the method itself, the people, the materials, and the environment.
- Process capability indices are simply a means of indicating **the variability of a process relative to the product specification tolerance**.

Calculation of process capability indices **estimates the short-term variations** within the process. This short-term is the period over which the process remains relatively stable. However, we know that processes do not remain stable for all time, and therefore we need to allow within the specified tolerance limits for:

- some movement of the mean
- detection of changes of the mean
- possible changes in the scatter (range)
- detection of changes in the scatter
- possible complications of non-normal distributions

This is the oldest index **based on a ratio of the mean range of samples to the tolerance band**. In order to avoid the production of defective material, the specification width must be greater than the process variation: $2T \geq 6\sigma$

We know that: $\sigma = \frac{\overline{R}}{d_n} = \frac{\text{Mean of Sample Ranges}}{\text{Hartley's Constant}}$ ⇨ Then, we obtain: $\frac{2T}{\overline{R}} \geq \frac{6}{d_n}$

$2T/\overline{R}$ is known as **Relative Precision Index (RPI)** and the value of $6/d_n$ is **the minimum RPI** to avoid the production of a material outside the specification limits. Moreover, RPI index **does not comment on the centring of a process** as it deals only with its relative spread or variation.

In our case study: $\text{Min. RPI} = \frac{6}{d_n} = \frac{6}{2.059} = 2.914$

If we are asked to produce rods within **± 10 mm** of the target length: ⇨ 2T = 20 mm ⇨ $\text{RPI} = \frac{2T}{\overline{R}} = \frac{20}{10.8} = 1.852$ ⇨ Reject Material

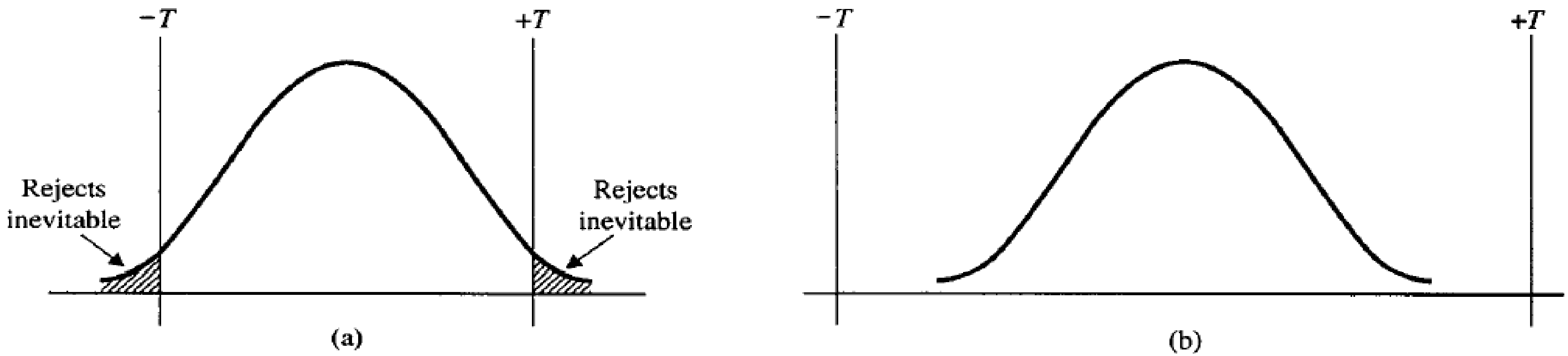
If the specified tolerances are widened to **± 20 mm**: ⇨ 2T = 40 mm ⇨ $\text{RPI} = \frac{2T}{\overline{R}} = \frac{40}{10.8} = 3.704$ ⇨ Avoid Rejecting Material

In order to manufacture within a specification, difference between USL and LSL must be less than total process variation. So, **a comparison of 6σ with (USL - LSL) or 2T** gives C_p index:

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{2T}{6\sigma}$$

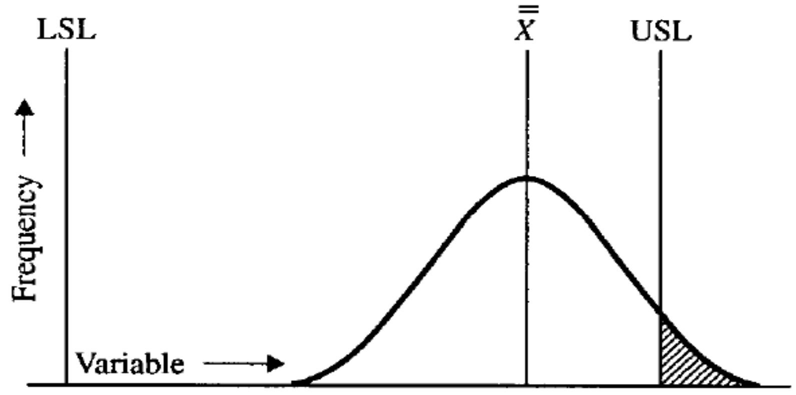
(a) $C_p < 1$ ⇨ Process variation is greater than tolerance band, so the process is **incapable**.

(b) $C_p \geq 1$ ⇨ For increasing values of C_p, the process becomes **increasingly capable**.



Similar with RPI, C_p index makes **no comment about the centring of the process**, which means that it is a simple comparison of total variation with tolerances.

Situation like this invalidate the use of C_p index, so there is a need for another index which takes account of **both the process variation and the centring**. Such an index is called **C_{pk} index**, which is widely accepted as a means of communicating process capability.



For USL and LSL limits, there are two C_{pk} values (**C_{pk_u}** and **C_{pk_l}**). These relate the difference between process mean and USL/LSL respectively, and **the lesser of these two** will be the value of C_{pk} index:

$$C_{pk} = \text{lesser of} \left(C_{pk_u} = \frac{USL - \bar{\bar{X}}}{3\sigma} \right) \text{or} \left(C_{pk_l} = \frac{\bar{\bar{X}} - LSL}{3\sigma} \right)$$

- **C_{pk} < 1** means that, considering the process variation and its centring, at least one of the tolerance limits is exceeded and **the process is incapable**.
- As in the case of C_p, **increasing values of C_{pk} correspond to increasing capability**.
- It may be possible to **increase C_{pk} value by centring the process** so that its mean value coincides with the mid-specification (target).
- Comparison of C_p and C_{pk} shows **no difference if the process is centred** on the target.
- C_{pk} can be used when there is only one specification limit, but C_p cannot be used in such cases.